Long-Wire Antennas

A Physical Picture of Rhombics and Vs

BY WALTER VAN B. ROBERTS,* W3CHO

THE COMPLETE analysis of the operation of long-wire antennas usually involves rather complicated mathematics, so it is interesting that some of the main results of such an analysis can be obtained in a relatively easy and simple manner which has the incidental advantage of providing a physical picture of their operation.

The present treatment starts with a simple long wire terminated nonreflectively so that purely traveling waves slide along it. These are waves of current, but they travel at the same speed along the wire as do radio waves in space so that both kinds of waves can be represented in the same way on a diagram.

Fig. 1 shows a long wire in free space set at an angle θ with respect to the direction in which transmission is desired. The diagram is in the

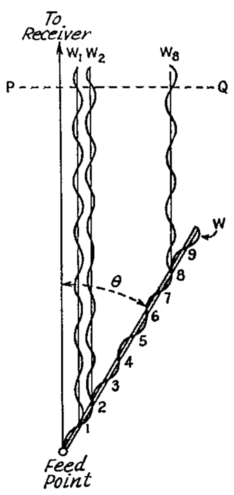

Fig. 1 **— How radiations from individual elements of a long wire add up along a given direction.**

nature of an instantaneous photograph showing current waves W along the wire and three sample radio waves, W_1, W_2, and W_8, which have been radiated from points 1, 2 and 8 on the wire and are on their way to the desired receiving point. All the various waves add up at the receiving location to produce a resultant field, but they must be added vectorially because they are not all in the same phase when they arrive. By drawing a line PQ across the waves it can be seen, for example, that W_2 is a little out of phase with W_1.

Fig. 2 shows how to find the vector sum, S, of the nine waves emitted from the nine points marked 1 to 9 on the wire. First a vector marked W_1 is laid off to represent wave W_1, then at the end of W_1 we draw a vector W_2 with just enough change of direction to correspond to the phase

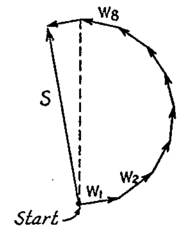

Fig. 2 **— Vector diagram showing the addition of field components from different sections of the radiator at a point in space.**

difference between W_1 and W_2. This process is continued until all the waves have been represented. The sum of all the vectors is the line from the starting point to the head of the last vector and is marked S. (If we had divided the wire up into a great many points instead of just nine, Fig. 2 would have looked like part of a smooth circle.)

It will be seen from Fig. 2 that S would be slightly greater if we had left off the ninth little vector. In other words, the wire is too long to produce the maximum signal possible in the desired direction. On the other hand, if the wire had been cut off at, say point 7, the sum S would again be less than the maximum possible. But it will also be noticed that the wire can be considerably longer or shorter than the optimum length without very much reduction in S, which fact accounts for the wide frequency range of operation of rhombics, for example.

Fig. 2 shows that the maximum value of S occurs when the waves from the two ends of the wire are 180 degrees out of phase. This happens when the wire is a half-wave longer than its projection along the line to the receiver[1] so that

• There are a few simple trig formulas and vector diagrams here, but don't let them frighten you. Essentially, the article is an easily-followed exposition of the principles underlying long-wire antennas. It will help you to visualize and understand what goes on, and why, in rhombics and Vs.

*155 Hodge Road, Princeton, N. J.

[1] Exactly the same condition determines the best length of a microwave horn for a given flare angle; also the diameter of the first Fresnel zone in wave propagation studies.

we have the equation $L = \frac{\lambda}{2} + L \cos\theta$ to tell us the optimum length of wire for a given value of θ. This equation may be rewritten thus

$$L = \frac{\lambda/2}{1 - \cos\theta} \qquad (1)$$

We have now developed a formula giving the best length of wire to use for a given angle θ, but what is the best value of θ to use? A plausible argument to determine this is as follows: Let us suppose that the wire is always divided into the same large number of parts so that the radiation from each part is represented by the individual vectors of a diagram such as Fig. 2. Obviously, the longer each vector is the greater S will be, assuming that the total wire length is always made to satisfy Equation 1. Now the radiation from each part of the wire in the desired direction is proportional to the length of the part multiplied by the sine of θ. Hence the field at the receiver will depend on $L \sin\theta$. Putting in the value of L given by Equation 1, the field is therefore determined by the quantity $\frac{\sin\theta}{1 - \cos\theta}$. It turns out that this quantity does not have a maximum value for any value of θ but continues getting bigger the smaller θ is made. Hence all we can say is that θ should be made as small as possible considering that L becomes very large as θ is made very small, which fact puts practical limits on the reduction of θ.

Ground Reflection

But in all the foregoing we have been talking about a wire in free space, which is not the usual condition. Usually the wire is stretched horizontally over ground. If the ground acts as a good reflector its action is to reinforce the radiation along a certain elevation angle which depends on the antenna height. This elevation angle will be figured later but for the present we will simply assume that there is such an angle and call it Δ. Referring to Fig. 1 again and considering it as a plan view of the antenna, we will now figure out all over again what is the best length of wire — but this time we want to know the best length for sending signals not directly toward the receiver but at an angle Δ above this line. Radiation is reinforced at this vertical angle in any case so we might as well design the antenna to work best in the same direction.

The recalculation happens to be very simple because Equation 1 tells us the optimum length in terms of the angle between the wire and the desired direction of transmission, so all we have to do is to consider the θ in Equation 1 as being replaced by the angle between the wire and a line elevated by the angle Δ *above* the direction to the receiver. By reference to the formulas of spherical trigonometry it will be found that the cosine of this new angle is simply the product $\cos\theta \cos\Delta$, where θ has the same meaning as in the previous discussion; that is, it is the horizontal angle between the wire and the direct line to the receiver. Hence Equation 1 becomes

$$L = \frac{\lambda/2}{1 - \cos\theta\cos\Delta} \qquad (2)$$

Again we have found how to make the length optimum for a given value of θ but do not know what is the best value of θ. To find out, we apply the same argument as before and this time find that the field at the elevation angle Δ is proportional to

$$\frac{\sin\theta}{1 - \cos\theta\cos\Delta}$$

This time there is a best value for θ, namely

$$\theta = \Delta. \qquad (3)$$

Equations 2 and 3 give definite values to L and θ which result in radiating the maximum possible signal at elevation angle Δ. All we need now is to know how high the wire should be so that reflection from the ground will reinforce signals transmitted at the elevation angle Δ.

Each of the individual waves from the wire is reflected from ground exactly like the waves from an ordinary dipole, so that the angle of reinforcement of radiation from the long wire will be the familiar angle that applies to horizontal dipoles. For the sake of completeness the derivation of this angle is shown in Fig. 3. The object is to find the elevation angle at which the direct and reflected rays are in phase. One way to do this is to know that horizontally-polarized waves are reversed in phase by reflection and then find what angle makes the path of the reflected ray a half-wave longer than that of the direct ray. The other way is to replace the earth by the image of the dipole, which is of opposite polarity to the dipole, and find what angle makes the path from the image a half-wave longer than that from the dipole. Either method gives the result:

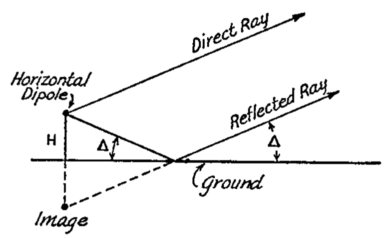

***Fig. 3* — Reflection from ground.**

$$H = \frac{\lambda}{4 \sin\Delta} \qquad (4)$$

This last equation supplies the finishing touch to the design of a long-wire antenna to give maximum signal at a given elevation angle. The equations may be rearranged in various ways by substituting values from one into another. For example, if we want to find best values of L, H and θ for transmission at a desired elevation angle Δ, the equations are more convenient thus:

$$H = \frac{\lambda}{4 \sin \Delta} \tag{5}$$

$$L = \frac{\lambda}{2 \sin^2 \Delta} \tag{6}$$

$$\theta = \Delta \tag{7}$$

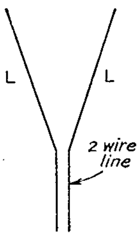

Fig. 4 — The V antenna.

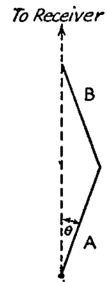

Fig. 5 — Combination of two long wires to form a half-rhombic antenna.

Vs and Rhombics

The wire has been shown slanting off to the right but it could just as well have been drawn off to the left except that the direction of the currents, or rather their crosswise components, would have appeared reversed as viewed from the receiving point. Thus if we use both right and left wires at once and drive them in push-pull their signals will add and we have a "V" antenna of optimum design, as shown in Fig. 4. Furthermore, Equations 5, 6 and 7 also give the optimum design of a rhombic whose sides are each of length L with an angle 2θ between sides at the feed point. That this is so will require a little more demonstration.

In Fig. 5 the wire A is chosen in accordance with Equations 5, 6 and 7 and hence radiations from its various parts add up to an exact semicircle in the diagram of Fig. 2. Now if wire B were added to A without a change of direction, the vector diagram of the radiations from the parts of B would simply continue around the circle in Fig. 2 to come back to the starting point, making the resultant S shrink back to nothing. But actually wire B is reversed in slope with respect to the line of transmission so that all the little vectors representing its radiations are reversed. Following the procedure for vector addition this results in the diagram of Fig. 6, which has the sum S twice as large as that from the single wire. If we now add two more wires, C and D, together with a two-wire line for feed and a terminating resistor R connecting the far ends of both sides, the result is the complete rhombic of Fig. 7.

There is one more thing we can deduce without much trouble, namely the power gain of the V and the rhombic as compared to the single wire of Fig. 1. When we add another wire to form a V the impedance presented to the source is doubled since the two wires are in effect fed in series, so that twice as much power is required to produce a given current in the two wires as in one alone. But the field at the receiving point also is doubled, and doubling the field is equivalent to quadrupling the power. Thus the V has a power gain of 2 over the single wire. Now adding elements B and C to the V to form a rhombic does not change the input impedance because there is no reflected wave, so the doubling of the receiver field that is thus produced is equivalent to quadrupling the power without any increase in actual input. Thus the rhombic has a power gain of 4 over the V. These results might be stated as follows: Each wire added to the single wire of Fig. 1 gives a power gain of 2.

Power Gain

Calculating the gain of a rhombic compared to a half-wave dipole is beyond the scope of the present qualitative sort of treatment, but an approximate value can be obtained from the known input resistance of the rhombic, which, when properly terminated and designed for maximum performance, is about 720 ohms. (Not that

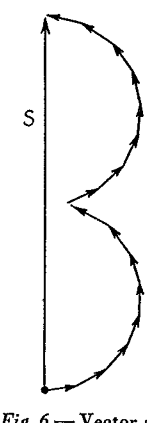

Fig. 6 — Vector addition of field components from the antenna shown in Fig. 5.

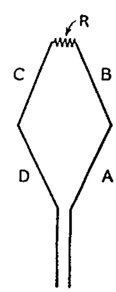

Fig. 7 — The rhombic antenna.

the last two digits are significant, but 720 is just ten times the input resistance of a half-wave dipole, making it a convenient figure to use.) If then we assume that the impedance of the rhombic is ten times that of a dipole, it will require ten times the power input to produce the same current in the rhombic as in a dipole. But the current in the rhombic is much more effective than the same current in a dipole. We can figure the relative fields set up in the desired direction by the same current in the two antennas by figuring their effective lengths, the term "effective length" being here defined as the length of wire that would be required to produce the observed signal at the receiver if the current were uniform and of the same phase throughout the wire, and the wire were crosswise to the line to the receiver. In the case of the half-wave dipole the current is in the same phase all along the wire but it is not of uniform strength, so that the effective length is $\frac{\lambda}{\pi}$. In the long-wire traveling-wave antenna the current is approximately uniform[2] but the varying phase of the waves received from different parts of the wire makes the resultant (see Fig. 2) only the diameter of a circle instead of the numerical sum of all the vectors, which is a semi-circumference. In other words, phase differences reduce the effective length by the factor $\frac{2}{\pi}$. Also, the long wire is not crosswise to the emitted beam so its effective length is further reduced by the factor $\sin A$ where A is the angle between the beam and the wire. Thus the effective length is $\frac{2}{\pi} L \sin A$. But we can use Equations 5, 6 and 7 to get rid of L, whence effective length is

$$\frac{\lambda}{\pi} \cdot \frac{\sin A}{\sin^2 \Delta}$$

In the rhombic the effective length is four times this value because there are four wires "pulling together" so that the effective length of the rhombic is

$$4 \frac{\sin A}{\sin^2 \Delta}$$

times that of a half-wave dipole and hence the fields produced by the same current are in that proportion. The ratio of the powers, being the square of the ratio of the fields, is

$$16 \frac{\sin^2 A}{\sin^4 \Delta}$$

But remembering that it takes ten times as much power to get the same current into the rhombic as into the dipole, the actual power gain is $1.6 \frac{\sin^2 A}{\sin^4 \Delta}$. The angle A can be eliminated from this expression since we found previously that $\cos A = \cos\theta \cos\Delta$ and by Equations 5, 6 and 7, this gives us

$$\cos A = \cos^2\Delta \text{ or } \sin^2 A = 1 - \cos^4\Delta$$
$$= (1 - \cos^2\Delta)(1 + \cos^2\Delta) = \sin^2\Delta\,(2 - \sin^2\Delta).$$

Thus, finally, the power gain is

$$\frac{3.2}{\sin^2\Delta} - 1.6$$

To see how this checks up assume $\Delta = 14\frac{1}{2}°$, which makes $\sin \Delta = \frac{1}{4}$. Then the gain is (3.2 x 16) − 1.6 or about 50, which is about 17 db. This value checks very closely with the value given by A. E. Harper of the Bell Telephone Laboratories.[3]

The present discussion is not intended to be used as the basis for the design of actual antennas or even for the calculation of their performance, because it only treats the case of the design for maximum possible output. In practice the dimensions can be economized considerably with very little loss in performance. For instance, in a detailed treatment of the rhombic[4] it is shown that there are even some advantages in reducing the lengths of the sides to 74% of the value given by equations 5, 6 and 7, the values of θ and H being unchanged from those given by the equations. The objective here is to give a physical picture of the operation of long wires and the relationship between different long-wire antennas so that more detailed treatments may be read with better understanding. The method employed may also be extended to determine the directions of other "lobes" of radiation from the long wire.

One final note: if the equations of this article are compared with equations in other treatments of rhombics a certain confusion may arise with respect to the angle θ. Other treatments usually deal with an angle which they call the "tilt angle" and which is 90 degrees minus θ. Of course it makes no difference which angle is used so long as we know what we are talking about, but the angle θ seemed the more natural one to use in the present derivations.

---

[2] The assumption has been made throughout this treatment that the current strength is uniform all along the wires. Obviously this cannot be true or else the entire input power would be delivered to the terminating resistor. However, the assumption appears close enough to the truth to permit reasoning to conclusions that are sufficiently accurate for the present purposes.

[3] Equation 19, page 59, *Rhombic Antenna Design*, by A. E. Harper, published by D. Van Nostrand Co., New York.

[4] Bruce, Beck and Lowry, "Horizontal Rhombic Antennas," *Proc. I.R.E.*, January, 1935.